Research Article

Introduction of Store Brands Considering Product Cost and Shelf Space Opportunity Cost

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This paper studies the introduction of store brands (SBs) when the product cost, shelf space opportunity cost, and baseline sales are taken into consideration. We construct a Stackelberg model in which one retailer, acting as the leader, sells a national brand (NB) and its SB and maximizes the category profit by allocating shelf space and determining the prices for the SB and NB products. Meanwhile, an NB manufacturer, acting as the follower, maximizes its profit based on the decisions of the retailer. Our results demonstrate that the product cost of the SB (NB) and the shelf space opportunity cost are the dominating factors that determine the optimal pricing strategy. If the two costs are low, then the optimal pricing strategy is the me-too strategy (competitive strategy); otherwise, the optimal pricing strategy is the differentiation strategy. There exists a threshold of the product cost, shelf space opportunity cost, and baseline sales to decide the pricing strategy and introduction of SB.

1. Introduction

Store brands (SBs) account for 14% of total retail sales in US supermarkets, and their share ranges from 20% to 45% of total retail sales in the UK, Belgium, Germany, Spain, and France [1]. Why are retailers eager to introduce SBs? SBs are the exclusive brands for which the retailer is responsible for shelf placement, pricing, quality, packaging, and promotion. In contrast to a national brand (NB), which is provided by the manufacturer, a retailer's SB product is entirely and independently controlled by the retailer, including research and development, design, sales, and market management. Retailers expand their market share by introducing SB products to extend their product line and to meet demands in different market segments [2, 3]. Moreover, retailers develop their SB products to compete with other retailers and enhance consumer loyalty [4]. Retailers also introduce SBs into the original sales category to obtain profits from their sales and, more importantly, to leverage negotiation power with the manufacturer [5]. Furthermore, Groznik and Heese [6] demonstrate that retailers are in a position to gain competitive advantages in the supply chain and increase profits by introducing SB products. Lamey et al. [7] also reveal that retailers are more

likely to increase their SBs market share when the economy is suffering and shrinks.

However, according to Nielsen's SBs report (the data come from Nielsen global private label report November 2014), the share of SBs in the Asia-Pacific market is generally low, and China's market share of SBs is only 1–3%. Hence, the SB market potential is extremely large and therefore attractive to growing numbers of retailers in China. Increasing numbers of retailers, including international brand retailers such as Walmart, ALDI, and Lidl and domestic brand retailers in China such as Lianhua, Vanguard, and Wumart, have begun to introduce SB products. ALDI opened an online store on TMall on Mar. 20th 2017. In ALDI supermarkets, most products are SBs, and generally, for each given category, there is a maximum of two other brands. ALDI's main competitor, Lidl supermarket, which is famous as an SB retailer in Germany, opened an online store in China on Sep. 28th 2017. Interestingly, the two retailers entered China's market via online stores.

Usually, the retailer develops its own SB products (see Figure 1). Manufacturer A sells its NB to the distributor at product price c, and the distributor delivers the NB to retailers at wholesale price w, and then the retailer sells the NB
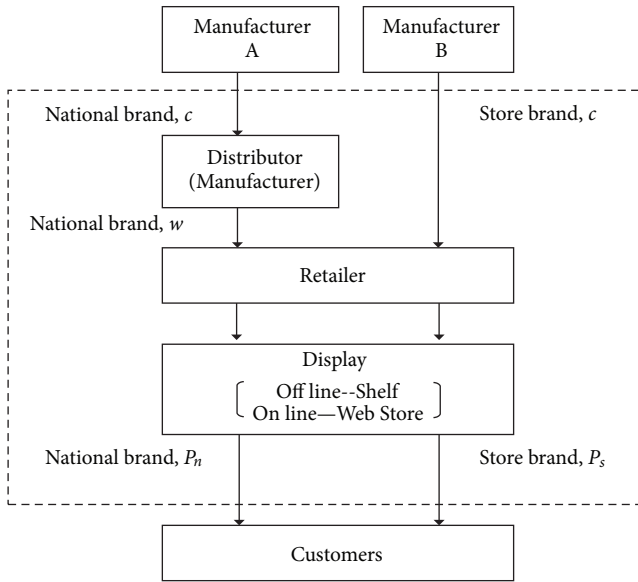
FIGURE 1: Supply chain structure.

products at retail prices P_n to the consumers through displays offline (shelf) or online (web store). Now, ALDI, as a powerful retailer, obtains the same quality of goods directly from manufacturer B, and it wants the product itself without any brand premium. Then, it sells these goods as SB products at retail prices P_s to consumers through the same displays. This is why ALDI's product quality is as good as that of any other retailer, while its prices are not high as others'. Therefore, it is natural to consider the retailer's purchasing cost and the display method as the dominant factors that affect retailers' profits and price positioning strategies.

Motivated by these issues, we propose the following research questions:

(1) Which factors will affect the price positioning strategy of a powerful retailer that is a leader and has the power to introduce SBs?

(2) What is a suitable price positioning strategy for a dominant retailer following the entry of an SB?

(3) Who will benefit from the different price positioning strategies?

To answer these questions, in this paper, we propose a Stackelberg model involving an NB manufacturer (this is the distributor in Figure 1) and a retailer selling its SB and the NB. The decision variables include the following: the wholesale price for the manufacturer, the retail prices of both brands, and the shelf space that the retailer allocates to each brand. We assume that the retailer is the leader in the Stackelberg game, and we characterize the resulting equilibrium in terms of price, shelf space, and profit for both players.

Generally, there are three-tiered SBs: economy SBs, standard SBs, and premium SBs (PSBs). Consumers generally perceive SBs to be lower quality and higher risk products. Geyskens and Steenkamp [8] note that initially retailers provide low-quality and low-cost SB products mainly as

substitutes for NB products. Furthermore, standard SBs imitate the quality of leading NB products for slightly lower prices. With the improvement in the quality of SBs, PSBs have become increasingly common in retail stores. Seenivasan et al. [9] report that "store brands have also gained in consumer esteem, with almost 77% of American consumers considering them to be as good as or better than national brands". The quality of some SB products has caught up to that of NB products, but SB pricing is usually lower than NB pricing. Nenycz-Thiel and Romaniuk [10] and Hara and Matsubayashi [11] show that the quality of some PSB products has caught up with that of NB products. In response to the introduction of SBs, manufacturers of NBs are searching for ways to expand their business and help retailers introduce their SB products. Hara and Matsubayashi [11] find that, in essence, some NB manufacturers have gradually become original equipment manufacturers (OEMs) of premium SBs. In this paper, we study the introduction of standard SBs.

To the best of our knowledge, no previous papers have studied the impact of product costs and shelf space opportunity costs on the entry of SBs. We fill the gap by proposing pricing strategy games between a retailer and a manufacturer when the retailer introduces SBs. There are two streams of literature, those on product cost and shelf space opportunity cost, that relate to the present research.

The frameworks employed in previous papers typically assume that the manufacturer who provides the SB product to the retailer does not play any strategic role, and thus they set the retailer's purchasing cost of the private brand at zero (see [12, 13]). However, in a recent contribution, Fang et al. [14] study a wholesale price contract between an NB supplier and retailer, and they consider the cost per unit quality (CPUQ), which can determine whether the retailer can introduce the SB and whether the supplier can affect and deter its introduction. In another recent work, Mai et al. [15] study an extended warranty as a means of coordinating the quality decisions for SB products. They consider the unit repair cost and unit production cost, which ensure that the product has a zero probability of failure during the extended warranty period. In contrast, our research shows that the product cost is the dominant factor that affects the price positioning strategy in the introduction of SBs by a powerful retailer.

The Stackelberg equilibrium solution will be adopted in this work. Amrouche and Zaccour [13] and Li et al. [16] study the shelf space allocation and pricing decisions in the marketing channel by applying static and dynamic games. Kurtuluş and Toktay [17] construct a supply chain with two manufacturers and one retailer and study a three-stage sequential dynamic game. They demonstrate that a retailer, acting as the leader in the supply chain, can use category management and categorize shelf space to control the intensity of competition between manufacturers. However, there they do not consider the impact of SBs or shelf space effects in the demand function. Kuo and Yang [18] develop a competitive shelf space model for NBs versus SBs based on Kurtuluş and Toktay's settings and find that if the cross-price effect is not too large,

the retailer should position its SB's quality closer to that of the NB. Kuo and Yang consider the shelf space opportunity cost in operation and channel conflict, but they do not consider product cost.

In retailing, the shelf space allocation problem is crucial and has been studied by both operations research and marketing scholars for years. Corstjens and Doyle [19] develop a model to address the shelf space allocation problem. Bultez and Naert [20] and Drèze et al. [21] confirm that shelf space has a positive effect on a retailer's sales and profitability. Irion et al. [22] develop a shelf space allocation optimization model that combines essential in-store costs and considers space- and cross-elasticities to study shelf space management. Valenzuela et al. [23] propose that consumers hold vertical schemas that higher is better on shelves and that more expensive products should be placed higher on a display than cheaper products. They test whether retailer shelf space layouts reflect consumer beliefs and illustrate that consumers' beliefs about shelf space layouts are not always reflected in the real marketplace. These studies all focus on the shelf space allocation of general products; however, they do not consider SBs or analyze the pricing issue. However, the competition for shelf space is prevalent in supermarkets, especially for new product introductions. Drèze et al. [21] demonstrate that retailers want to maximize category sales and profits and must allocate a certain amount of shelf space to do so. Moreover, manufacturers want to maximize the sales and profits of their NBs and therefore always want more and better space to be allocated to their NBs. Thus, retailers often earn a positive profit margin on each product they sell in addition to collecting the slotting fees, given that their role goes beyond shelf space leasing [24]. Because the slotting fee includes not only shelf space leasing but also logistics, merchandising, and promotion, among other services, shelf space is so scarce that manufacturers have to provide retailers with slotting fees to secure shelf space for their SBs. Our research assumes that the shelf space opportunity cost represents a slotting fee that is a dominant factor affecting the introduction and price positioning strategy of SBs.

In contrast to all of the above research streams, our paper discusses the store brand entry problem under varying product cost and SB opportunity scenarios. Our results provide guidance for retailers regarding marketing strategies under different product cost, shelf space opportunity, and baseline sales settings. This is one of our contributions to the existing literature.

The remainder of the paper is organized as follows: In Section 2, we construct an economic profit model for the supply chain under study. In Section 3, we derive the Stackelberg equilibrium. In Section 4, we seek the optimal pricing strategy by conducting a numerical study with different scenarios. In Section 5, we conclude the paper.

2. The Model

We consider a two-stage supply chain that consists of one retailer and one manufacturer. The manufacturer provides one product in a given category and sells it to consumers through the retailer. The retailer maximizes profit by allocating shelf space to each brand. We normalize the total shelf space available for each category to one. S denotes the share of this space that is dedicated to the SB, and $S > 0$. We assume that the total shelf space is allocated; thus, the share of the NB is $1 - S$ (see [13]). We assume that the demand for each brand depends on the exposure each receives, as measured by shelf space and the price of each brand. Here, we assume the NB baseline sales are normalized to one and the baseline sales of the SB are captured by the parameter $\alpha_s \in (0, 1)$ (see [12]). The demands of the two products are as follows:

$$D_n = \frac{1}{1 + \alpha_s} \left((1 + \alpha_s)(1 - S) + \psi (P_s - P_n) - P_n \right) \quad (1)$$

$$D_s = \frac{1}{1 + \alpha_s} \left((1 + \alpha_s) S + \psi (P_n - P_s) - P_s \right) \quad (2)$$

where D_n and D_s represent the demand for the NB and for the SB, respectively. $1 + \alpha_s$ represents the total baseline sales (potential market) of the NB and SB [6]. $\psi \in (0, 1)$ denotes the cross-price competition between the NB and SB. P_n and P_s are the reference prices for each, respectively, which is a common assumption in the literature (see [12, 17, 25]). In addition, the demand for each brand increases in its proportion of shelf space. The rationale is that if a product has more shelf space, the probability of being noticed, perceived, and selected by the consumer will increase (see [26–29]). Furthermore, many studies demonstrate that each brand's demand increases in the competing brand's price and decreases in its own price (see [12, 18]). According to (1) and (2), the marginal price effect on demand depends on the shelf space allocated to the brand. This specification has been used extensively in the literature (see [30, 31]).

In addition, c is the unit cost. As in Nenycz-Thiel and Romaniuk [10] and Hara and Matsubayashi [11], we assume that the costs of the SB and the NB are equal. To simplify the computation, we assume that there is no significant quality differentiation between the SB and NB, and the prices of the two products satisfy

$$P_s = \gamma P_n, \quad 0 < \gamma < 1 \quad (3)$$

where P_s and P_n represent the retail prices of the SB and NB, respectively. γ is the price difference coefficient. Parameter c represents the cost of the SB and NB. Assuming that the manufacturer and the retailer are profit maximizers, their objectives are as follows:

$$\max \quad \Pi_M = (w - c) D_n \quad (4)$$

$$\max \quad \Pi_R = D_s (P_s - c) + m D_n - \frac{k S^2}{2} \quad (5)$$

where Π_M and Π_R represent the profit of the manufacturer and of the retailer, respectively, and D_n and D_s are given by (1) and (2), respectively. w represents the wholesale price, and

m represents the unit markup from selling unit NB; therefore, $P_n = w + m$. Because shelf space is a scarce resource, space allocated to one product means relinquishing profits from another product. If the shelf spaces of NBs are occupied by the SB, there exists the loss of the opportunity cost for the retailer. That means the retailer will forgo the slotting fee for the new NB from manufacturer. Assume that the retailer incurs a shelf space cost, i.e., the opportunity cost k of the shelf space, where $k > 0$. The shelf space proportion, S, is a continuous endogenous variable. The assumption is standard in economics (see [17, 32]).

In this paper, we assume that the retailer is the leader and the manufacturer is the follower. The sequence of events is as follows: The retailer (leader) first announces its marketing strategy, including the unit markup value m and shelf space proportion S. The manufacturer reacts to this information by deciding the wholesale price w.

3. Stackelberg Equilibrium

To determine the reaction function of the manufacturer to the retailer's unit markup value m and shelf space proportion S, we must solve the following optimization problem. First, we consider the manufacturer's problem. Substituting (1) and (3) into (4) yields

$$\Pi_M$$
$$= \frac{(w - c)(\psi(\gamma(m + w) - m - w) - m + (1 - S)(\alpha_s + 1) - w)}{\alpha_s + 1}. \tag{6}$$

First-order conditions are

$$\frac{d\Pi_M}{dw} = 0 \iff$$

$$w(m, S) \tag{7}$$

$$= \frac{c((\gamma - 1)\psi - 1) - \gamma m\psi + m\psi + m + (S - 1)\alpha_s + S - 1}{2(\gamma - 1)\psi - 2}$$

and

$$\frac{d^2\Pi_M}{dw^2} = \frac{2((\gamma - 1)\psi - 1)}{\alpha_s + 1} < 0. \tag{8}$$

Next, we address the retailer's optimization problem. Substituting (7) into (5) yields

$$\Pi_R = \frac{1}{4}\left(-2kS^2 + R_1 - R_2(R_3 + R_4)\right) \tag{9}$$

where

$$R_1 = \frac{2m((\gamma - 1)\psi(c + m) - c - m - (S - 1)\alpha_s - S + 1)}{\alpha_s + 1}$$

$$R_2 = \frac{c(\gamma - 2)((\gamma - 1)\psi - 1) + \gamma((\gamma - 1)m\psi - m + S - 1) + \gamma(S - 1)\alpha_s}{(-\gamma\psi + \psi + 1)^2(\alpha_s + 1)} \tag{10}$$

$$R_3 = ((\gamma - 1)\psi + \gamma)((\gamma - 1)\psi(c + m) - c - m - 1)$$

$$R_4 = \alpha_s(-\gamma(\psi + 1) + S(\gamma(-\psi) + \gamma + \psi + 2) + \psi) + S(\gamma(-\psi) + \gamma + \psi + 2).$$

We obtain the following results.

Theorem 1. *If the price difference coefficient γ and cross-price competition coefficient ψ satisfy $\gamma > (K(K + \psi - 1) - 4\psi + 1)/K\psi$, then the retailer's profit function Π_R is jointly concave in m and S, where*

$$K = \left[-4\psi^2 + 2\sqrt{4\psi^4 + 4\psi^3 - \psi^2} + 6\psi - 1\right]^{1/3}. \tag{11}$$

Proof. The first- and second-order derivatives of Π_R with respect to m and S are as follows:

$$\frac{\partial^2\Pi_R(m, S)}{\partial m^2} = -\frac{\gamma^2 + (1 - \gamma)(2 - \gamma)\psi + 2}{2(\alpha_s + 1)} < 0. \tag{12}$$

$$\frac{\partial^2\Pi_R(m, S)}{\partial S^2} = -\frac{\gamma(\gamma(1 - \psi) + \psi + 2)(\alpha_s + 1)}{2((\gamma - 1)\psi - 1)^2} - 1 \tag{13}$$
$$< 0$$

$$\frac{\partial\Pi_R(m, S)}{\partial m\partial S} = \frac{-\gamma(\gamma + \psi + 1) + \psi + 1}{2(\gamma - 1)\psi - 2} \tag{14}$$

$$\frac{\partial\Pi_R(m, S)}{\partial S\partial m} = \frac{-\gamma(\gamma + \psi + 1) + \psi + 1}{2(\gamma - 1)\psi - 2} \tag{15}$$

The Hessian matrix can be formed as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 \Pi_R (m, S)}{\partial m^2} & \dfrac{\partial^2 \Pi_R (m, S)}{\partial m \partial S} \\[2mm] \dfrac{\partial^2 \Pi_R (m, S)}{\partial S \partial m} & \dfrac{\partial^2 \Pi_R (m, S)}{\partial S^2} \end{bmatrix} = \begin{bmatrix} -\dfrac{2 + \gamma^2 + (2 - \gamma)(1 - \gamma)\psi}{2(1 + \alpha_s)} & \dfrac{1 + \psi - \gamma(1 + \gamma + \psi)}{-2 + 2(-1 + \gamma)\psi} \\[2mm] \dfrac{1 + \psi - \gamma(1 + \gamma + \psi)}{-2 + 2(-1 + \gamma)\psi} & -1 - \dfrac{\gamma[2 + \gamma(1 - \psi) + \psi](1 + \alpha_s)}{2(-1 + (-1 + \gamma)\psi)^2} \end{bmatrix} \tag{16}$$

$$\frac{\partial^2 \Pi_R (m, S)}{\partial m^2} = -\frac{2 + \gamma^2 + (2 - \gamma)(1 - \gamma)\psi}{2(1 + \alpha_s)} < 0.$$

Since $\alpha_s \in (0, 1), \psi \in (0, 1), \gamma > (K(K + \psi - 1) - 4\psi + 1)/K\psi$, $K = \sqrt[3]{2L - 4\psi^2 + 6\psi - 1}, L = \sqrt{4\psi^4 + 4\psi^3 - \psi^2}$, then

$$|H| = \frac{k(\gamma^2 + (\gamma - 2)(\gamma - 1)\psi + 2)}{2(\alpha_s + 1)} - \frac{3\gamma^2 + (\gamma - 1)^3 \psi + 6\gamma - 1}{4(\gamma - 1)\psi - 4} > 0. \tag{17}$$

H is a negative definite integral and the profit function is concave. Setting $\partial \Pi_R(m, S)/\partial m = 0, \partial \Pi_R(m, S)/\partial S = 0$ and letting (m^*, S^*) represent the solutions to the linear systems, then

$$m^* = \frac{m_1 + m_2 + m_3}{X} \tag{18}$$

$$S^* = \frac{S_1}{X} \tag{19}$$

where

$$m_1 = (\gamma + 1)(2(\gamma - 1)k(\psi + 1) - \gamma) - \gamma(\gamma + 1)\alpha_s^2$$

$$m_2 = \alpha_s \left(c(\gamma - 1)^3 \psi + 2c(\gamma(\gamma + 2) - 1) + 2(\gamma + 1) \right.$$

$$\left. \cdot ((\gamma - 1)k(\psi + 1) - \gamma) \right).$$

$$m_3 = c \left((\gamma - 1)^3 \psi + 2(\gamma(\gamma + 2) - 1) \right.$$

$$- 2k((\gamma - 1)\psi - 1)$$

$$\left. \cdot (\gamma((\gamma - 3)\psi + \gamma - 1) + 2\psi + 1) \right) \tag{20}$$

$$X = 2k((\gamma - 1)\psi - 1)(\gamma^2 + (\gamma - 2)(\gamma - 1)\psi + 2)$$

$$+ (-3\gamma^2 + (\gamma - 1)^3(-\psi) - 6\gamma + 1)(\alpha_s + 1)$$

$$S_1 = (\gamma + 1)(c(\gamma - 3)((\gamma - 1)\psi - 1) - \gamma(\psi + 4)$$

$$+ (-\gamma(\psi + 4) + \psi + 1)\alpha_s + \psi + 1)$$

Substituting m^* and S^* into (6) yields

$$w^* = \frac{w_1 - w_2 + w_3}{X} \tag{21}$$

where

$$w_1 = k \left(-2\gamma^2(\psi + 1) + c((\gamma - 1)\psi - 1) \right.$$

$$\cdot (\gamma(2\gamma(\psi + 1) - 6\psi - 1) + 4\psi + 3) + 3\gamma\psi - \psi$$

$$\left. - 1 \right)$$

$$w_2 = \alpha_s \left(c(\gamma - 1)^3 \psi + 2\gamma((c - 1)\gamma + 3c + 1) \right. \tag{22}$$

$$\left. + k(\gamma(2\gamma(\psi + 1) - 3\psi) + \psi + 1) \right)$$

$$w_3 = -c(\gamma - 1)^3 \psi + \gamma(-2c(\gamma + 3) + \gamma - 1) + \gamma(\gamma - 1)\alpha_s^2$$

and then

$$P_n^* = \frac{P_{n1} + P_{n2}}{X} \tag{23}$$

$$P_s^* = \gamma P_n^* \tag{24}$$

where

$$P_{n1} = -(\gamma - 1)k\psi(\gamma c + c + 3) + c(\gamma + 1)(k + 2)$$

$$+ 2\gamma + 3k \tag{25}$$

$$P_{n2} = \alpha_s (2c(\gamma + 1) + 4\gamma + 3k(-\gamma\psi + \psi + 1) + 2\gamma\alpha_s)$$

$$D_n^* = \frac{D_{n1}(D_{n2} + D_{n3})}{(\alpha_s + 1)(Y1 - Y2)} \tag{26}$$

$$D_s^* = \frac{\alpha_s(D_{s1} + D_{s2}) + D_{s3} + D_{s4}}{(\alpha_s + 1)(Y1 - Y2)} \tag{27}$$

where

$$D_{n1} = (\gamma - 1)\psi - 1$$

$$D_{n2} = \alpha_s \left(c(\gamma^2 - 1) + 2(\gamma - 1)\gamma \right.$$

$$\left. - k(\gamma(2\gamma(\psi + 1) - 3\psi) + \psi + 1) + (\gamma - 1)\gamma\alpha_s \right)$$

$$D_{n3} = c(\gamma^2 + k(\gamma^2(-\psi) + \gamma + \psi + 1) - 1) + (\gamma - 1)$$

$$\cdot \gamma - k(2\gamma^2(\psi + 1) - 3\gamma\psi + \psi + 1)$$

$$D_{s1} = -c(\gamma + 1)((\gamma - 1)^2 \psi + \gamma + 3) + 3(\gamma - 1)^2$$

$$\cdot k\psi^2 + (\gamma - 1)^2(3k - 2)\psi$$

$$D_{s2} = 4\gamma^2 + 6\gamma - 3\gamma k - (\gamma(\gamma(\psi - 2) - 2\psi - 3) + \psi$$
$$+ 1)\alpha_s - 2$$

$$D_{s3} = 2\gamma^2 + 3\gamma + 3(\gamma - 1)^2 k\psi^2 + (\gamma - 1)^2(3k - 1)\psi$$
$$- 3\gamma k - 1$$

$$D_{s4} = c(\gamma + 1)(-\gamma + (\gamma - 1)^2 k\psi^2$$
$$+ (\gamma - 1)^2(k - 1)\psi - \gamma k - 3)$$

$$Y1 = (3\gamma^2 + (\gamma - 1)^3\psi + 6\gamma - 1)(\alpha_s + 1)$$

$$Y2 = 2k((\gamma - 1)\psi - 1)(\gamma^2 + (\gamma - 2)(\gamma - 1)\psi + 2)$$
$$\tag{28}$$

and thus

$$\Pi_M^* = (w^* - c)D_n^* \tag{29}$$

$$\Pi_R^* = (P_s^* - c)D_n^* + m^* D_n^* - \frac{1}{2}k(S^*)^2. \tag{30}$$

To determine whether the retailer can increase profit through the introduction of an SB, we need to consider the case when the retailer sells only the NB. We assume that the demand for the NB depends on the price of the NB if there is no SB. The following functional forms are assumed:

$$D_n = 1 - P_n. \tag{31}$$

We assume that $P_n = w + m$, and then we have the following optimization problem:

$$\max \quad \Pi_{Mo} = (w - c)D_n = (w - c)(1 - m - w) \tag{32}$$

$$\max \quad \Pi_{Ro} = mD_n = m(1 - m - w). \tag{33}$$

Using an analytical process that is similar to the Stackelberg equilibrium, we obtain $\Pi_{Ro} = (1/8)(c - 1)^2$, $\Pi_{Mo} = (1/16)(c - 1)^2$ and the final demand of the product category, $D_n = (1 - c)/4$. □

Proof. Please see the proof in Appendix A. □

4. Numerical Studies

The purpose of this section is to reveal the effects of the product cost and the baseline sales of SBs on profitability and shelf space allocation.

4.1. Scenario 1: Varying Product Cost of SBs. In this subsection, we will study the relationships between the product cost c and the parameters, such as the decision variables, demand, price, and profitability of SBs and NBs. Let $\alpha_s = 0.8$, $\psi = 0.8$, $k = 0.1$, and $\gamma = 0.7; 0.8; 0.9$.

Figure 2(a) shows that SB shelf space S and markup m decrease as product cost c increases, and conversely the wholesale price of NB w increases. In Figure 2(b), the demand

of SB, D_s, and total demand, D, decrease as product cost c increases; meanwhile, the demand of NB, D_n, increases. Figure 2(c) shows that product prices of both NBs and SBs increase as product cost c increases. That is to say, most of the decision variables and other parameters (except the demand for the NB) are sensitive to product cost c, and any slight change in the product cost results in a great change in the parameters (such as S, m, w, D, D_s).

In Figures 3 and 4, we aim to study (1) the relationship between the product cost c and the profit of the retailer and manufacturer when the SB is introduced, as well as (2) the profit of the retailer and manufacturer before and after the introduction of SB.

Figure 3(a) shows that the retailer's total profit decreases as product cost c increases. There exists a cost threshold $\bar{c} = 0.425$ (where $\alpha_s = 0.8$, $\psi = 0.8$, $k = 0.1$, and $\gamma = 0.7$ or 0.9). The retailer should use a different pricing strategy for different product costs. If the cost is less than the threshold, the me-too strategy is preferred; if the cost is larger than the threshold, the differentiation strategy can increase profit. In other words, when the product cost c is high, the retailer uses a differentiation strategy; however, the me-too strategy is better when the product cost c is low.

Figure 3(b) demonstrates the profit before and after the introduction of SB when c changes. It shows that the total profits of the retailer decrease as product cost c increases. There exists a cost threshold $\tilde{c} = 0.389$ (where $\alpha_s = 0.8$, $\psi = 0.8$, $k = 0.1$, and $\gamma = 0.8$) such that if the cost is less than the threshold, the introduction of the SB is profitable; if the cost is larger than the threshold, the introduction of the SB will not increase profit, and the retailer will not have enough incentive to introduce the SB. However, the retailer is less affected when differentiation strategies are used. Within the range $[0, \tilde{c}]$, the retailer uses differentiation strategies, and its total profit will reach a minimum value and then rise again. That is, differentiation strategies are used for the price decision, and the introduction of SB will bring more profits for the retailers, due to the existence of big price differential. This conclusion is different from the previous research; Sayman et al. [33] do not find the significant effect of price differential; in present paper, we demonstrate that the significant effect exists when considering the product cost.

Figure 4 demonstrates that the manufacturer's profits will be very low when the retailer introduces the SB. However, the manufacturer's total profits increase as product cost c increases. Furthermore, when the retailer introduces the SB and the product cost increases, the manufacturer's total profit is higher than before. However, this situation will not occur because it is not profitable for the retailer when parameter c is too high; the retailer, as a leader, will not introduce the SB in this interval. To put it differently, the retailer will prudently consider whether to introduce the SB.

4.2. Scenario 2: Varying Shelf Space Opportunity Cost of SBs. In this subsection, we will study the relationships between the shelf space opportunity cost of SB k and the parameters, such as decision variables, demand, price, and the profitability of SBs and NBs. Let $\alpha_s = 0.8$, $\psi = 0.8$, $c = 0.1$, and $\gamma = 0.7; 0.8; 0.9$.
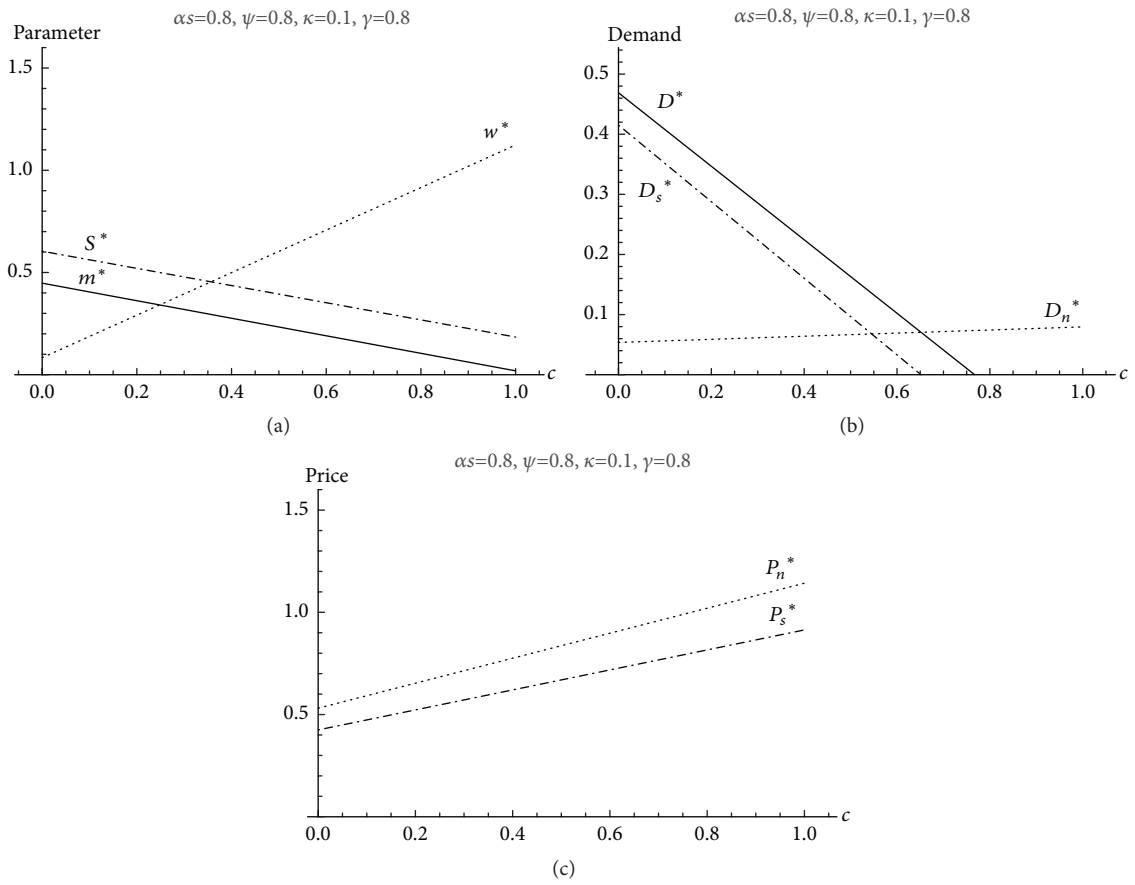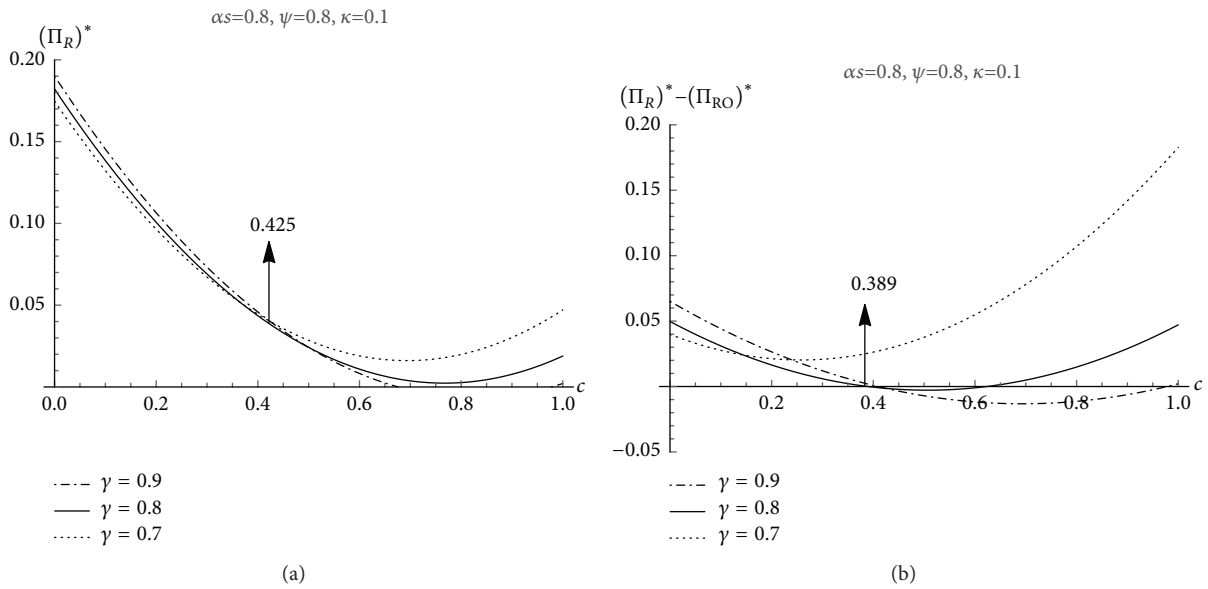
FIGURE 2: Variable results for different values of c.

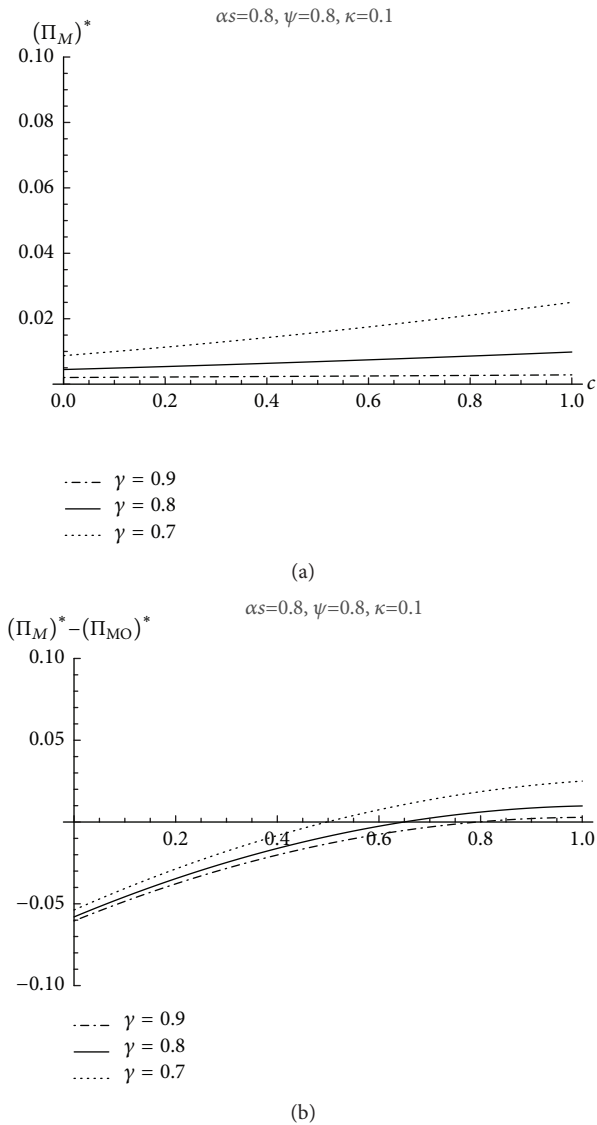FIGURE 3: Retailer's total profit for different values of c.

(a)

(b)

FIGURE 4: Manufacturer's total profit for different values of c.

The results in Figure 5(a) show that when in the high-competition situation ($\psi = 0.8$) with high baseline sales of SBs ($\alpha_s = 0.8$), increasing the opportunity cost parameter of shelf space k leads to a sharp decrease in the retailer's shelf space proportion S, a slow decrease in the unit markup value m, and a sharp increase in the wholesale price w. Figure 5(b) demonstrates that increasing the opportunity cost of shelf space k causes a sharp decrease in the retailer's demand for SBs but an increase in the manufacturer's demand for NBs and, subsequently, a sharp increase in the total demand for the product category. Figure 5(c) indicates a slow increase in the sales price for both SBs and NBs when k increases.

In Figures 6 and 7, we aim to study (1) the relationship between the opportunity cost of the shelf space k and the profit of retailer and manufacturer when the SB is introduced, as well as (2) the profit difference of the retailer and the manufacturer before and after the introduction of SB.

Figure 6(a) shows that the retailer's total profits decrease as the opportunity cost of the shelf space k increases. Meanwhile, the retailer should use a different pricing strategy when the opportunity cost takes a different value. Particularly, there exists a threshold $\bar{k} = 0.252$ (where $\alpha_s = 0.8$, $\psi = 0.8$, $c = 0.1$, and $\gamma = 0.7$ or 0.9), such that if the opportunity cost of the shelf space is less than the threshold, the retailer uses the me-too strategy; if the opportunity cost of the shelf space is larger than the threshold, a differentiation strategy is optimal.

Figure 6(b) visualizes the difference in profit before and after the introduction of the SB; the retailer's total profits decrease as opportunity cost parameter of shelf space k increases. There exists a threshold $\tilde{k} = 0.374$ (where $\alpha_s = 0.8$, $\psi = 0.8$, $c = 0.1$, and $\gamma = 0.8$). When the opportunity cost parameter k is high, the retailer uses a differentiation strategy; on the other hand, the competitive strategy (me-too strategy) is better when parameter k is low.
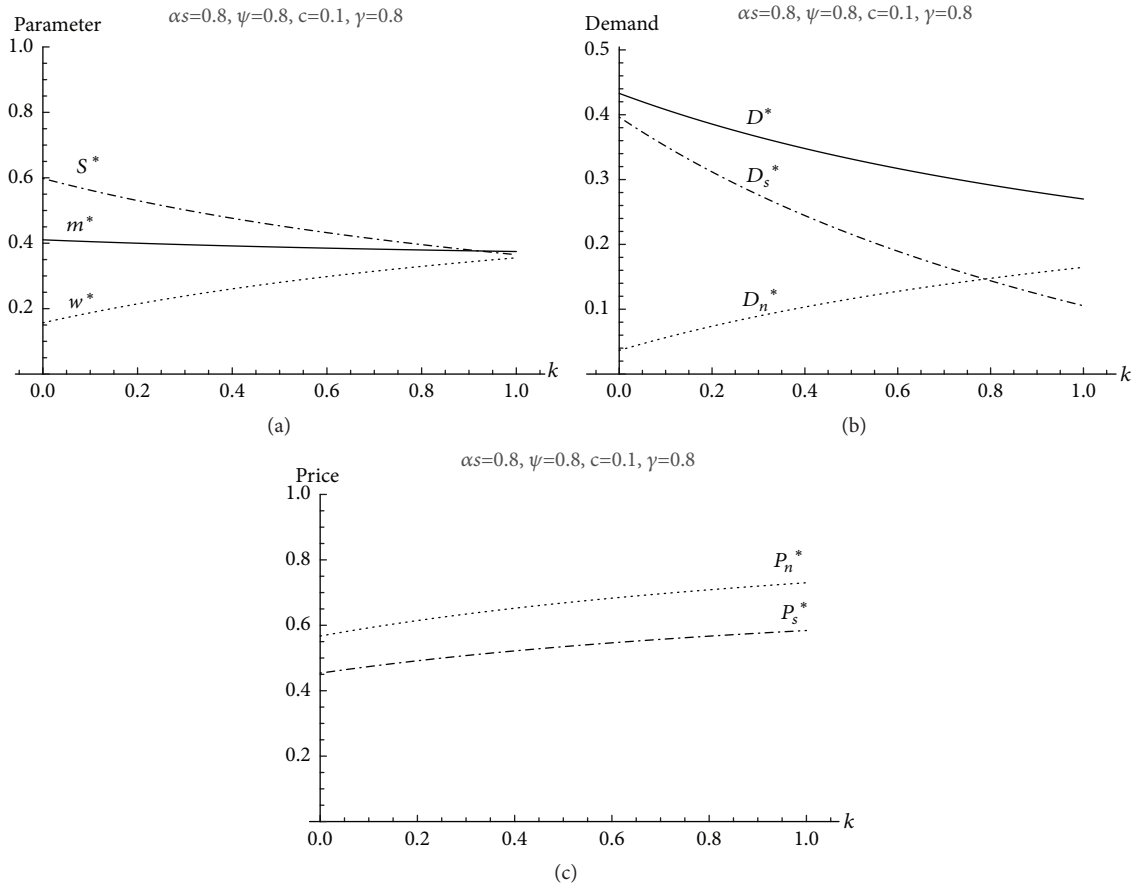
Figure 5: Variable results for different values of k.

Figure 7 demonstrates that the manufacturer's total profit increases as the opportunity cost of shelf space k increases. That is to say, when the retailer introduces SB, the manufacturer's total profit is higher than before. However, this situation will not occur because the retailer is not profitable when parameter k is high; in this interval, the retailer, as a leader, will not introduce the SB.

4.3. Scenario 3: Varying Baseline Sales of SBs. Let $\psi = 0.8$, $k = 0.8$, $c = 0.1$, and $\gamma = 0.8$; we draw the plots for the relationships between the baseline sales of SBs, α_s, and parameters such as the decision variables, demand, price, and profitability of SBs and NBs.

Figure 8(a) indicates that in a high-competition situation where $\psi = 0.8$ and the low-cost parameters $k = 0.1$, $c = 0.1$, and $\gamma = 0.8$, an increase in the baseline sales of SB α_s leads to a slow increase in the SB's proportion of retail shelf space, S, and a sharp increase in the unit markup value, m. Meanwhile, the manufacturer's wholesale price, w, increases slowly as baseline sales of the SB, α_s, increases. Figure 8(b) demonstrates that increasing the baseline sales of SBs α_s causes a sharp increase in the retailer's demand for SBs, a decrease in the manufacturer's demand for NBs, and, subsequently, an increase in the total demand for the product category. When the retailer, as a leader, introduces the SB, actual sales increase as the baseline sales increas.

That is to say, as the baseline sales of the SB increase, the retailer can increase the proportion of shelf space allocated to the SB; meanwhile, the retailer is better off because it can obtain more profit from the manufacturer's product. In this situation, the increase of the wholesale price can be perceived as compensation for the shelf space occupied by the SB, and the manufacturer deliberately increases the wholesale price to offset the manufacturer's loss from NB sales. Meanwhile, Figure 8 also demonstrates that as α_s increases, the retailer can increase the SB's proportion of shelf space. The actual demand of the SB increases more quickly than that of the shelf space, which aligns with the conclusion in Eisend [28], that a small increase in shelf space elasticity can also promote a rapid growth in product sales.

In Figure 9, we study the relationship between the retail price of SB (NB) and the baseline sales of the SB α_s. Figure 9(a) shows that in a high-competition situation where $\psi = 0.8$ and the low-cost parameters $k = 0.1$, $c = 0.1$, and $\gamma = 0.8$, the prices of both the SB and NB increase as the baseline sales of the SB α_s increase. Figure 9(b) demonstrates that when the retailer introduces SB and implements the me-too strategy (competitive strategy), the price of the NB is lower than the differentiation strategy, which aligns with the conclusion in Gabrielsen and Sørgard [34], that the introduction of SB leads to price concessions from the NB.
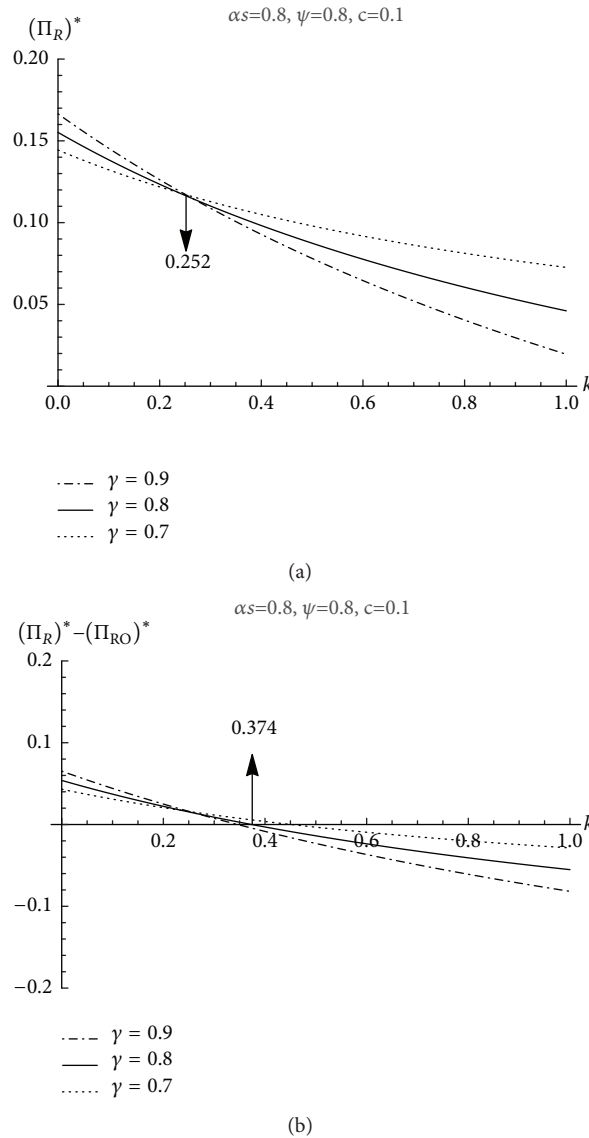
$\alpha s=0.8$, $\psi=0.8$, $c=0.1$

(a)

$\alpha s=0.8$, $\psi=0.8$, $c=0.1$

(b)

Figure 6: Retailer's total profit for different values of k.

Eventually, it will be beneficial for consumers to purchase the NB.

In Figure 10, we study the profitability of retailer in different marketing environments. Let $\psi = 0.8$, $k = 0.1; 0.35$, $c = 0.5; 0.1$, and $\gamma = 0.7; 0.8; 0.9$. As α_s increases, the retailer's total profit increases; however, there is a significant difference when the retailer uses different pricing strategies. (1) When c or k is small, the retailer uses the me-too strategy, and the profit of the retailer will increase. (2) When c or k is large, then the differentiation strategy (i.e., $\gamma = 0.7$) results in more profit than the me-too strategy (i.e., $\gamma = 0.9$).

In Figure 11, we study the difference in profit before and after the introduction of SB. The result shows that if retailer introduces the SB, there exists the threshold $\overline{\alpha_s} = 0.455$ (where $\psi = 0.8$, $k = 0.1$, $c = 0.1$, and $\gamma = 0.8$), such that (1) under the differentiation strategy, if α_s is greater than 0.492 (where $\psi = 0.8$, $k = 0.1$, $c = 0.1$, and $\gamma = 0.7$), it will be

profitable to introduce the SB; (2) under the me-too strategy, if α_s is greater than 0.414 (where $\psi = 0.8$, $k = 0.1$, $c = 0.1$, and $\gamma = 0.9$), it will be profitable to introduce the SB.

In Figure 12, we study the differences in profit before and after the introduction of SB. The result demonstrates that when the retailer, as the leader, introduces the SB, the manufacturer will gain little profit; however, a comparison of the profit before and after SB is introduced shows that the profit of the manufacturer reduces considerably. In other words, when the retailer is the leader, the introduction of the SB is detrimental to the manufacturer, and this result aligns with Kuo and Yang [18].

In addition, Figure 12(a) demonstrates that when the retailer uses the me-too strategy ($\gamma = 0.9$), as α_s increases, the manufacturer's profits gradually decrease. When the retailer uses the differentiation strategy ($\gamma = 0.7$), as α_s increases, the manufacturer's profits gradually increase. Thus, if the retailer,
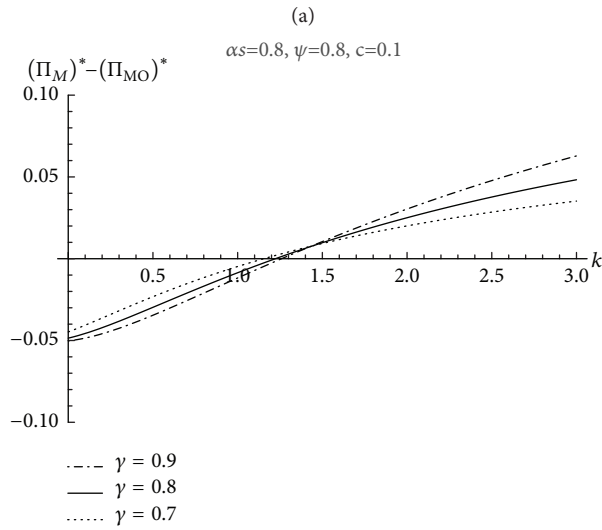
$(\Pi_M)^*$
$\alpha s=0.8,\ \psi=0.8,\ c=0.1$

$-\cdot-\ \gamma = 0.9$
$\underline{\quad\quad}\ \gamma = 0.8$
$\cdots\cdots\ \gamma = 0.7$

(a)

$\alpha s=0.8,\ \psi=0.8,\ c=0.1$
$(\Pi_M)^*-(\Pi_{MO})^*$

$-\cdot-\ \gamma = 0.9$
$\underline{\quad\quad}\ \gamma = 0.8$
$\cdots\cdots\ \gamma = 0.7$

(b)
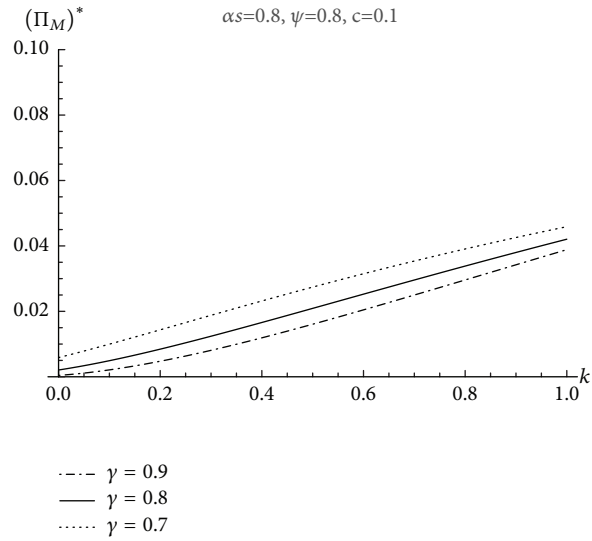
Figure 7: Manufacturer's total profit for different values of k.

$\psi=0.8,\ k=0.1,\ c=0.1,\ \gamma=0.8$
Parameter

$\psi=0.8,\ k=0.1,\ c=0.1,\ \gamma=0.8$
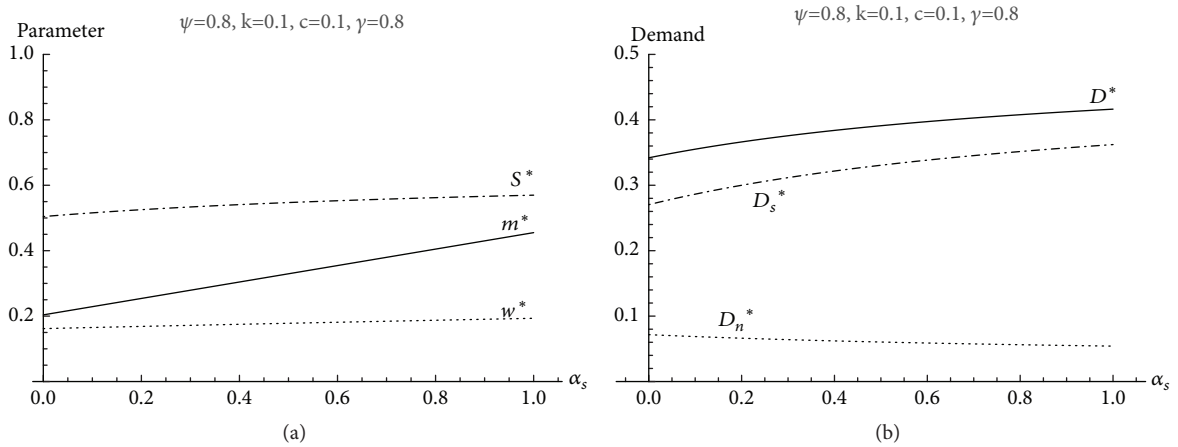Demand

(a)

(b)

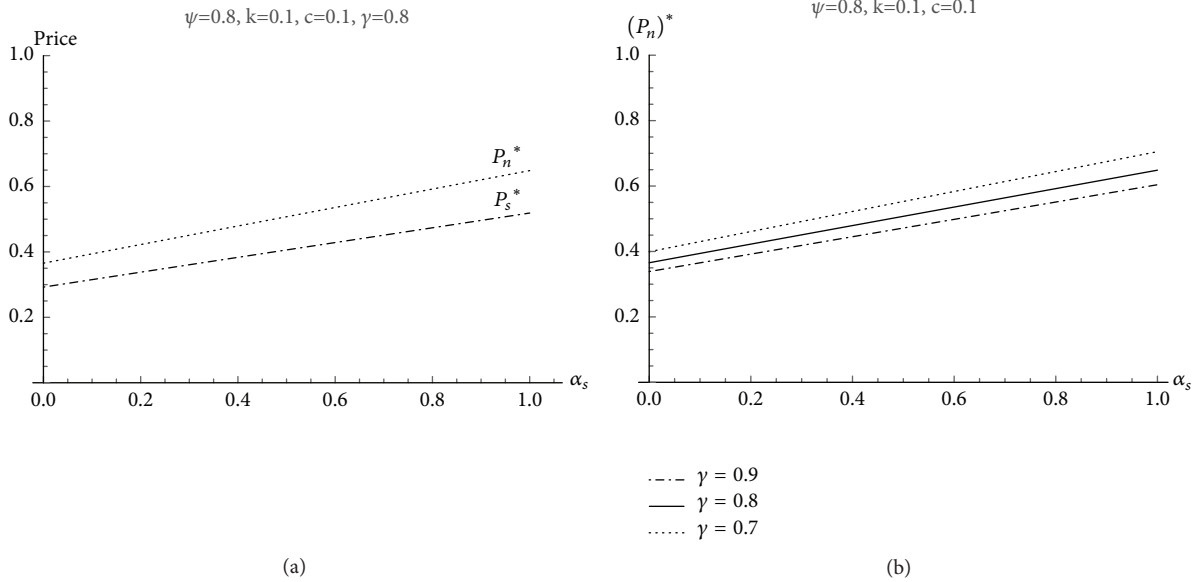Figure 8: Variable results for different values of α_s.

FIGURE 9: Price for different values of α_s.

as the leader, introduces the SB, the manufacturer is eager to increase its profit when the retailer adopts a differentiation strategy. Therefore, when the SB and the NB have roughly the same product quality, a differentiation strategy helps to cultivate consumers' preferences and to improve consumer loyalty to the SB. Furthermore, the differentiation strategy is more conducive to the introduction of other categories of the SB.

4.4. Scenario 4: Manufacturer as the Leader in the Supply Chain. Figure 13 demonstrates the results when the manufacturer is the leader (please see the proof in Appendix B). It indicates that in this case, although the shelf space proportion S is greater than 0 (where $\psi = 0.8$, $k = 0.1$, $c = 0.1$, and $\gamma = 0.9$), the actual demand for the SB is less than 0. That is, the SB should be introduced only when the retailer is the leader and has sufficient power. When the manufacturer is the leader in the supply chain, the retailer does not have an incentive to introduce the SB.

5. Conclusion

In this paper, we investigate the introduction of an SB product when the retailer is the supply chain leader. In particular, our aim is to answer the following questions: (1) What is the price positioning strategy of the SB—the differentiation or the me-too strategy—when the product cost and the shelf space opportunity cost are considered? (2) What are the factors that influence the pricing position of the retailer? (3) Who will benefit from the different price strategies? To answer these questions, this paper examines a two-echelon supply chain that consists of a manufacturer and a retailer. The retailer sells an NB product produced by the manufacturer and an SB

product. The retailer needs to determine the price markup of the NB, the price of the SB, and the shelf space allocated to the SB. The manufacturer needs to determine the wholesale price of the product. To this end, we formulate a Stackelberg game model in which the retailer is the leader and the manufacturer is the follower.

Our contribution is twofold. On the one hand, we prove the condition that an optimal solution exists. On the other hand, to distinguish the factors that influence the introduction and pricing position strategy of the SB, we conduct an experimental analysis of the parameters. Our results indicate that if both the product cost of the SB and the shelf space opportunity cost are low, then the optimal pricing strategy is the me-too strategy (competitive strategy). Otherwise, the optimal pricing strategy is the differentiation strategy.

To the best of our knowledge, previous papers have not studied the impact of product cost and shelf space opportunity cost on the entry of SBs. With regard to retailers, our findings have a number of managerial implications: (1) according to the numerical analysis, there is a significant effect of the price differential between the SB and NB; that is, an SB with a price positioned as close as possible to the NB price will not generate more profit for the retailer when the SB is a standard SB. This conclusion is different from those of previous research [33]. This is because our research considers the role of product cost, and we observe a significant effect. (2) There exist thresholds $\tilde{c}$ and $\tilde{k}$ of costs such that if the cost is less than the threshold, the introduction of the SB is profitable; if the cost is larger than the threshold, then the introduction of the SB will not increase profits, and the retailer will not have sufficient incentive to introduce the SB. According to our numerical analysis, the introduction of an SB and the optimal pricing strategy cannot be fully captured
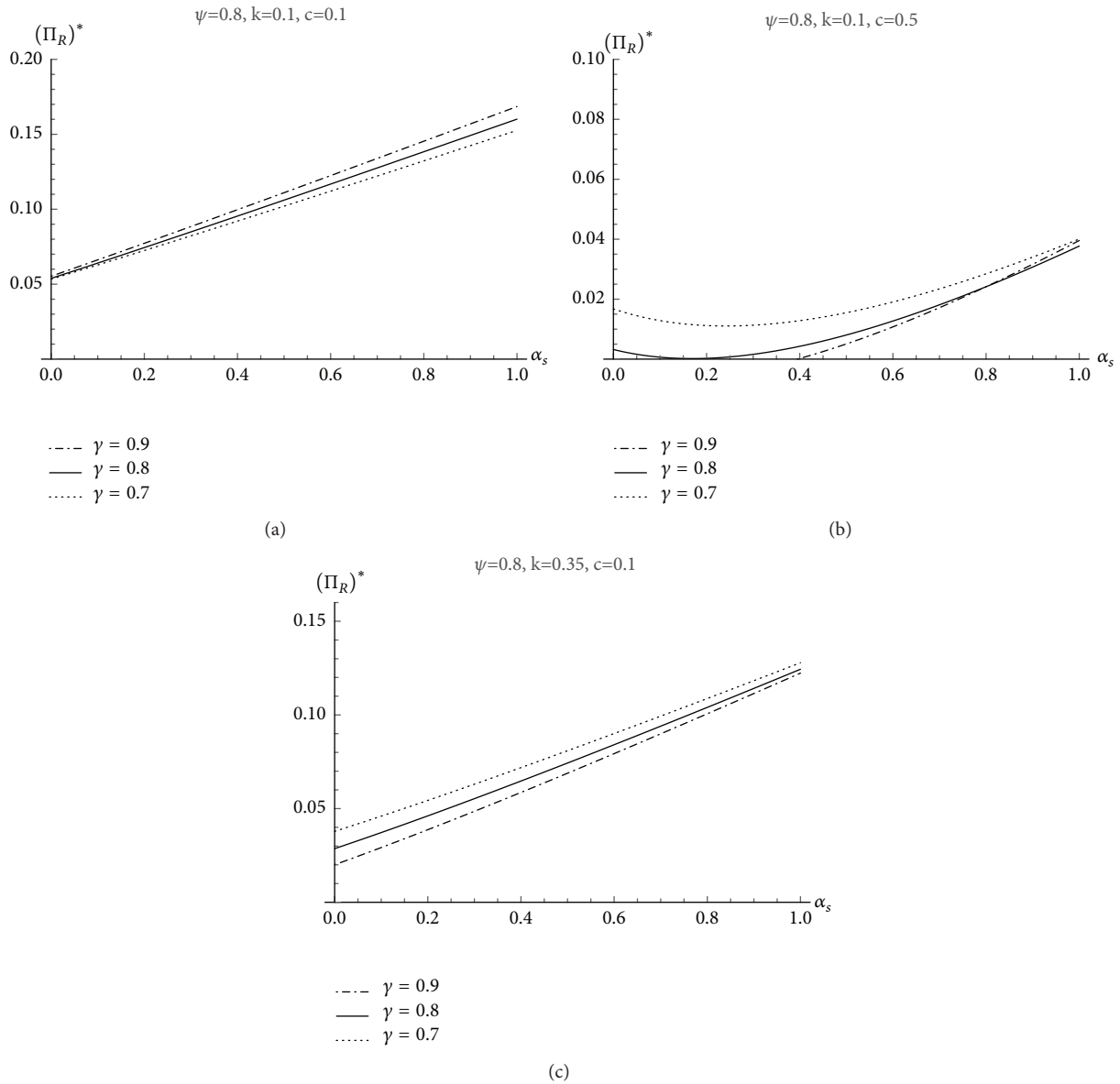
FIGURE 10: Retailer's total profit for different values of α_s.

by only one parameter. That is, the product cost and shelf space opportunity cost are the dominant factors affecting the introduction of an SB. The conclusion contrasts with the findings of previous research (see [12, 17]). This is because the effect of shelf space is reflected not only in the demand function but also in the profit function. (3) There also exists a threshold of baseline sales such that if the baseline sales of the SB are less than the threshold, the introduction of the SB will not increase profits, and the retailer will not have sufficient incentive to introduce the SB; if the baseline sales are larger than the threshold, the introduction of the SB is profitable. (4) The numerical analyses also show that the manufacturer is better off when the retailer adopts a differentiation strategy

and enlarges the price differential. However, the retailer's pricing strategies are dependent on the product costs and shelf opportunity cost. In addition, the retailer uses a me-too strategy; in this case, the prices of both the NB and the SB are lower, and consumers will therefore be better off when they purchase either the NB or the SB.

This study has several shortcomings that are worthy of further investigation in the future. First, we assume that the product cost for each brand is the same. In reality, most products do not have the same cost. Therefore, it would be interesting to extend our model to include different costs, Second, our model does not consider competition between retailers or between manufacturers. In fact, with
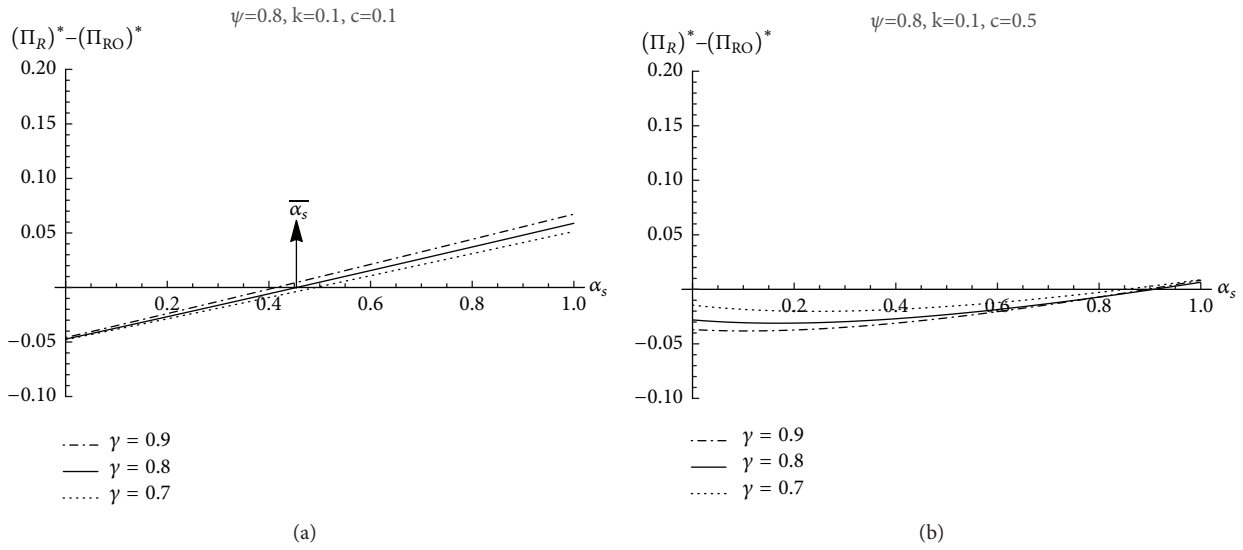
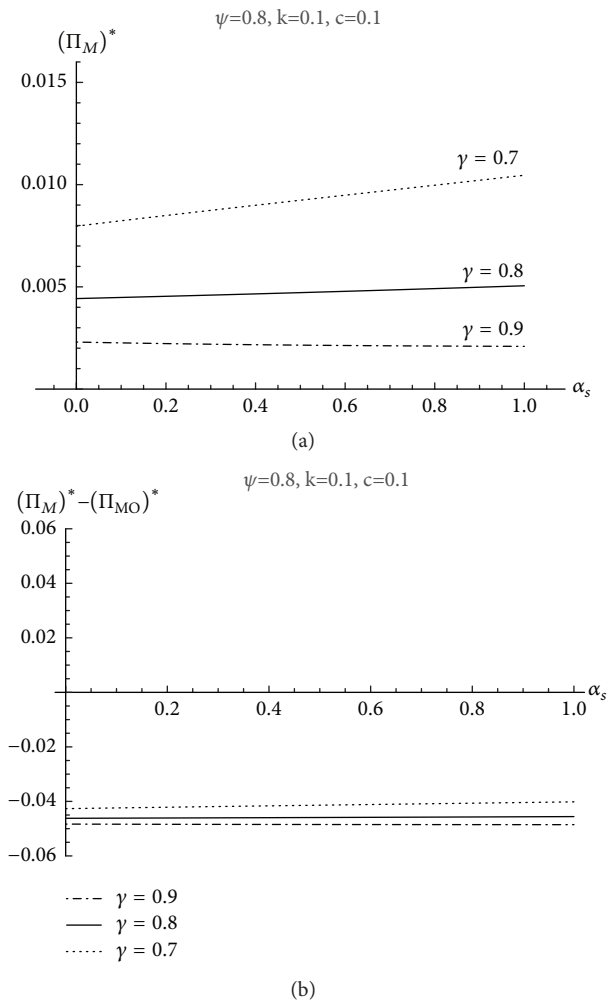FIGURE 11: Comparison between the periods before and after the introduction of the SB in α_s.

FIGURE 12: Comparison between the periods before and after the introduction of the SB in α_s.
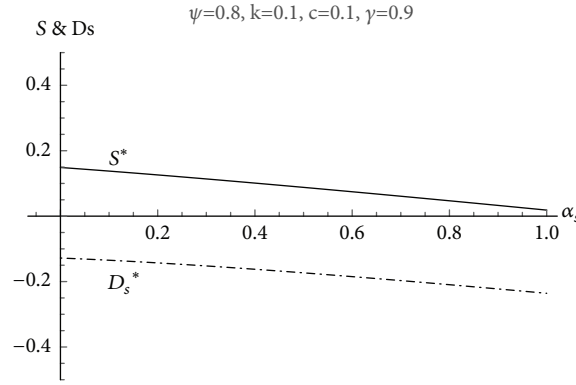
FIGURE 13: Demand and shelf space proportion for the SB for different values of α_s.

the improvement of SB quality, retailers have their own SBs, and SB competition needs to be considered even though the resulting model would certainly be difficult to analyze.

Appendix

A. Retailer Sells Only the NB

The scenario before introducing the SB: We assume that the demand for the NB depends on the price of the NB if there is no SB. The following functional forms are assumed:

$$D_n = 1 - P_n \tag{A.1}$$

$$P_n = m + w \tag{A.2}$$

$$\Pi_{MO} = (w - c) D_n \tag{A.3}$$

$$\Pi_{RO} = m D_n. \tag{A.4}$$

First, we consider the manufacturer's problem. The first-order optimality conditions are

$$\frac{d\Pi_{MO}}{dw} = 0 \iff$$

$$w(m) = \frac{1}{2}(c - m + 1). \tag{A.5}$$

Substituting (A.1), (A.2), and (A.5) into (A.4) yields

$$\Pi_{RO} = -\frac{1}{2}m(c + m - 1). \tag{A.6}$$

The concavity of Π_{RO} and the first-order condition yields

$$\frac{d\Pi_{RO}}{dm} = \frac{1}{2}(-c - 2m + 1) = 0 \iff$$

$$m^* = \frac{1-c}{2}. \tag{A.7}$$

Then, substituting (A.7) into (A.5) yields

$$w^* = \frac{1}{4}(3c + 1). \tag{A.8}$$

Therefore,

$$P_n{}^* = w^* + m^* = \frac{1}{4}(c + 3) \tag{A.9}$$

$$D_n{}^* = 1 - P_n{}^* = \frac{1-c}{4} \tag{A.10}$$

$$\Pi_{RO}{}^* = m^*(D_n)^* = \frac{1}{8}(c - 1)^2 \tag{A.11}$$

$$\Pi_{MO}{}^* = w^*(D_n)^* = \frac{1}{16}(c - 1)^2. \tag{A.12}$$

B. Manufacturer Stackelberg

The manufacturer is powerful and is a leader in the specific product category.

$$\Pi_R = \frac{(\gamma(m+w) - c)(-((\gamma-1)\psi + \gamma)(m+w) + S\alpha_s + S) + m((\gamma-1)\psi(m+w) - m - (S-1)(\alpha_s+1) - w)}{\alpha_s + 1} - \frac{kS^2}{2} \tag{B.1}$$

We obtain the following results.

Theorem B.1. *If the baseline sales of the SB α_s, shelf space cost k, and cross-price competition coefficient ψ satisfy $2k\psi > \alpha_s + 1$, then the retailer's profit function Π_R is jointly concave in m and S.*

Proof. The first- and second-order derivatives of Π_R with respect to m and S are as follows:

$$\frac{\partial^2 \Pi_R(m, S)}{\partial m^2} = -\frac{2\left(\gamma^2 + (\gamma - 1)^2 \psi + 1\right)}{\alpha_s + 1} < 0 \qquad (B.2)$$

$$\frac{\partial^2 \Pi_R(m, S)}{\partial S^2} = -k < 0 \qquad (B.3)$$

$$\frac{\partial \Pi_R(m, S)}{\partial m \partial S} = \gamma - 1 \qquad (B.4)$$

$$\frac{\partial \Pi_R(m, S)}{\partial S \partial m} = \gamma - 1. \qquad (B.5)$$

The Hessian matrix can be formed as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 \Pi_R(m, S)}{\partial m^2} & \dfrac{\partial^2 \Pi_R(m, S)}{\partial m \partial S} \\[3mm] \dfrac{\partial^2 \Pi_R(m, S)}{\partial S \partial m} & \dfrac{\partial^2 \Pi_R(m, S)}{\partial S^2} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{2\left(1 + \gamma^2 + (-1 + \gamma)^2 \psi\right)}{1 + \alpha_s} & -1 + \gamma \\[3mm] -1 + \gamma & -k \end{bmatrix} \qquad (B.6)$$

$$\frac{\partial^2 \Pi_R(m, S)}{\partial m^2} = -\frac{2\left(1 + \gamma^2 + (-1 + \gamma)^2 \psi\right)}{1 + \alpha_s} < 0.$$

Since $\alpha_s \in (0, 1)$, $\psi \in (0, 1)$, $k > 0$, $2k\psi > \alpha_s + 1$, then

$$|H| = \frac{2k\left(\gamma^2 + (\gamma - 1)^2 \psi + 1\right)}{\alpha_s + 1} - (\gamma - 1)^2 > 0. \qquad (B.7)$$

H is a negative definite integral and the profit function is concave. By the concavity of Π_R, the first-order condition yields

$$\frac{\partial \Pi_R}{\partial m} = 0 \iff$$

$$m(w, S) = \frac{c\gamma\psi + c\gamma - c\psi + \alpha_s((\gamma - 1)S + 1) + (\gamma - 1)S - 2\gamma^2 w\psi - 2\gamma^2 w + 3\gamma w\psi - w\psi - w + 1}{2\left(\gamma^2(\psi + 1) - 2\gamma\psi + \psi + 1\right)} \qquad (B.8)$$

$$\frac{\partial \Pi_R}{\partial S} = 0 \iff$$

$$S(w, m) = \frac{-c + (\gamma - 1)m + \gamma w}{k}. \qquad (B.9)$$

Thus,

$$m(w) = -\frac{-c\gamma + c\gamma k + \alpha_s(-\gamma c + c + k + (\gamma - 1)\gamma w) + (\gamma - 1)k\psi(c - 2\gamma w + w) + c - 2\gamma^2 kw - kw + k + \gamma^2 w - \gamma w}{(\gamma - 1)^2 - 2k\left(\gamma^2 + (\gamma - 1)^2\psi + 1\right) + (\gamma - 1)^2\alpha_s} \qquad (B.10)$$

$$S(w) = -\frac{-c\left(\gamma^2 + (\gamma - 1)^2\psi + \gamma + 2\right) + \gamma + (\gamma - 1)\alpha_s + w\left(2\gamma^2 + (\gamma - 1)^2\psi + \gamma + 1\right) - 1}{(\gamma - 1)^2 - 2k\left(\gamma^2 + (\gamma - 1)^2\psi + 1\right) + (\gamma - 1)^2\alpha_s}. \qquad (B.11)$$

Then,

$$\Pi_M = \frac{(w - c)\left(\psi(\gamma(m + w) - m - w) - m + (1 - S)(\alpha_s + 1) - w\right)}{\alpha_s + 1}. \qquad (B.12)$$

Substituting (B.10) and (B.11) into (B.12) yields

$$\Pi_M = \frac{(M_1 + M_2 + M_3)(c - w)}{M_4} \qquad (B.13)$$

where

$$M_1 = \left(c\left((\gamma - 1)^2 k\psi^2 + (\gamma - 1)^2 k\psi + \gamma(\gamma - k + 2) + 1\right)\right)$$

$$M_2 = \alpha_s\left(c(\gamma + 1)^2 + k\left(2\gamma^2(\psi + 1) - 3\gamma\psi + \psi + 1\right) - (\gamma - 1)\gamma\alpha_s - 2(\gamma(\gamma + \gamma w + w - 1))\right)$$

$$M_3 = \gamma + k\left(2\gamma^2\left(\psi + 1\right) - 3\gamma\psi - w\left(-\gamma\psi + \psi + 1\right)^2\right.$$
$$\left. + \psi + 1\right) - \gamma\left(\gamma + 2\left(\gamma + 1\right)w\right)$$
$$M_4 = \left(\alpha_s + 1\right)\left(\left(\gamma - 1\right)^2\right.$$
$$\left. - 2\left(k\left(\gamma^2 + \left(\gamma - 1\right)^2\psi + 1\right)\right) + \left(\gamma - 1\right)^2\alpha_s\right).$$
$$\text{(B.14)}$$

The first-order optimality conditions are

$$w^* = \frac{w_1 + w_2 + w_3}{w_4} \tag{B.15}$$

where

$$w_1 = c\left(\left(\gamma + 1\right)\left(3\gamma + 1\right)\right.$$
$$\left. + \left(\gamma - 1\right)\left(\left(\gamma - 1\right)\psi - 1\right)\left(k\left(2\psi + 1\right)\right)\right)$$
$$w_2 = \alpha_s\left(\left(3\gamma + 1\right)\left(c\left(\gamma + 1\right)\right) + 2\gamma\right.$$
$$+ 2\gamma^2\left(k\psi + k - 1\right) - 3\gamma k\psi + k\psi + k$$
$$\left. - \left(\gamma - 1\right)\gamma\alpha_s\right)$$
$$w_3 = \gamma + \gamma^2\left(2\left(k\left(\psi + 1\right)\right) - 1\right) - 3\gamma k\psi + k\psi + k$$
$$w_4 = 2\left(2\gamma\left(\gamma + 1\right) + k\left(-\gamma\psi + \psi + 1\right)^2\right.$$
$$\left. + 2\left(\gamma\left(\gamma + 1\right)\right)\alpha_s\right). \tag{B.16}$$

Substituting (B.15) into (B.10) and (B.11) yields

$$m^* = -\frac{c\left(m_1 + \left(k^2\left(m_2 - m_3\right)\right)\right) + \alpha_s\left(c * m_7 + \left(\gamma\left(m_{11} - m_{10}\right)\right)\alpha_s - 3 * m_6 + m_8 + m_9\right) + m_4 + m_5 - m_6}{m_{12} * m_{13}} \tag{B.17}$$

where

$$m_1 = 3\left(\gamma\left(\gamma + 1\right)\right)\left(\gamma - 1\right)^2$$
$$+ k\left(\left(\gamma\left(-5\gamma^3 + 18\gamma - 16\right)\right)\psi + \gamma\left(-6\gamma^3 - 5\gamma^2\right.\right.$$
$$\left.\left. + \gamma - 7\right) + 2\left(\gamma - 1\right)^4\psi^2 + 3\psi + 1\right)$$
$$m_2 = -2\left(\gamma\left(\gamma\left(\left(\gamma - 5\right)\gamma + 8\right) - 7\right)\right)\psi + \gamma\left(2\left(\gamma - 1\right)\right.$$
$$\left. \cdot \gamma + 3\right) - 6\psi - 1$$
$$m_3 = 3\left(\gamma - 1\right)^2\left(\gamma\left(2\gamma - 3\right) + 3\right)\psi^2 - 4\left(\gamma - 1\right)^4\psi^3$$
$$m_4 = k^2\left(-4\gamma^4\left(\psi + 1\right)^2 + 12\gamma^3\left(\psi\left(\psi + 1\right)\right)\right.$$
$$- \gamma^2\left(\psi\left(11\psi + 8\right) + 4\right) + 2\gamma\left(\psi\left(\psi + 1\right)\right) + \left(\psi\right.$$
$$\left.\left. + 1\right)^2\right)$$
$$m_5 = 2k\left(\gamma\left(\gamma\left(\gamma\left(2\gamma\left(\psi + 1\right) - 5\psi - 2\right) + 4\psi + 3\right)\right.\right.$$
$$\left.\left. - \psi + 1\right)\right)$$
$$m_6 = \left(\gamma - 1\right)^2\gamma^2$$
$$m_7 = 6\left(\gamma\left(\gamma + 1\right)\right)\left(\gamma - 1\right)^2$$
$$+ k\left(\left(\gamma\left(-5\gamma^3 + 18\gamma - 16\right)\right)\psi + \gamma\left(-6\gamma^3 - 5\gamma^2\right.\right.$$
$$\left.\left. + \gamma - 7\right) + 2\left(\gamma - 1\right)^4\psi^2 + 3\psi + 1\right)$$
$$m_8 = k^2\left(-4\gamma^4\left(\psi + 1\right)^2 + 12\gamma^3\left(\psi\left(\psi + 1\right)\right)\right.$$
$$- \gamma^2\left(\psi\left(11\psi + 8\right) + 4\right) + 2\gamma\left(\psi\left(\psi + 1\right)\right) + \left(\psi\right.$$
$$\left.\left. + 1\right)^2\right)$$

$$m_9 = 4k\left(\gamma\left(\gamma\left(\gamma\left(2\gamma\left(\psi + 1\right) - 5\psi - 2\right) + 4\psi + 3\right)\right.\right.$$
$$\left.\left. - \psi + 1\right)\right)$$
$$m_{10} = 3c\left(\gamma + 1\right)\left(\gamma - 1\right)^2 + 3\gamma\left(\gamma - 1\right)^2 - \gamma\left(\gamma - 1\right)^2$$
$$\cdot \alpha_s$$
$$m_{11} = 2\left(k\left(\gamma\left(\gamma\left(2\gamma\left(\psi + 1\right) - 5\psi - 2\right) + 4\psi + 3\right)\right.\right.$$
$$\left.\left. - \psi + 1\right)\right)$$
$$m_{12} = 2\left(\left(\gamma - 1\right)^2 - 2\left(k\left(\gamma^2 + \left(\gamma - 1\right)^2\psi + 1\right)\right)\right.$$
$$\left. + \left(\gamma - 1\right)^2\alpha_s\right)$$
$$m_{13} = 2\gamma\left(\gamma + 1\right) + k\left(-\gamma\psi + \psi + 1\right)^2 + 2\left(\gamma\left(\gamma\right.\right.$$
$$\left.\left. + 1\right)\right)\alpha_s \tag{B.18}$$

$$S^* = \frac{S_1}{S_2} - \frac{S_3\left(S_4 + S_5\right)}{S_6 S_7} \tag{B.19}$$

where

$$S_1 = c\left(\gamma^2 + \left(\gamma - 1\right)^2\psi + \gamma + 2\right) - \gamma - \left(\gamma - 1\right)\alpha_s$$
$$+ 1$$
$$S_2 = \left(\gamma - 1\right)^2 - 2\left(k\left(\gamma^2 + \left(\gamma - 1\right)^2\psi + 1\right)\right) + \left(\gamma\right.$$
$$\left. - 1\right)^2\alpha_s$$
$$S_3 = \gamma^2\left(\psi + 2\right) - 2\gamma\psi + \gamma + \psi + 1$$

$$S_4 = c \, ((\gamma + 1)(3\gamma + 1)$$
$$+ (\gamma - 1)((\gamma - 1)\psi - 1)(k(2\psi + 1))) + \gamma$$
$$+ \gamma^2 \, (2(k(\psi + 1)) - 1) - 3\gamma k \psi + k\psi + k$$

$$S_5 = \alpha_s \, ((3\gamma + 1)(c(\gamma + 1)) + 2\gamma$$
$$+ 2\gamma^2 \, (k\psi + k - 1) - 3\gamma k \psi + k\psi + k$$
$$- (\gamma - 1)\gamma \alpha_s)$$

$$S_6 = 2 \, ((\gamma - 1)^2 - 2(k(\gamma^2 + (\gamma - 1)^2 \psi + 1))$$
$$+ (\gamma - 1)^2 \alpha_s)$$

$$S_7 = 2\gamma(\gamma + 1) + k(-\gamma\psi + \psi + 1)^2 + 2(\gamma(\gamma + 1))$$
$$\cdot \alpha_s.$$

$$\text{(B.20)}$$

Thus,

$$P_n^* = w^* + m^* \tag{B.21}$$

$$P_s^* = \gamma P_n^* \tag{B.22}$$

$$D_n^*$$
$$= \frac{1}{1 + \alpha_s} \, ((1 + \alpha_s)(1 - S^*) - P_n^* + \psi(P_s^* - P_n^*)) \tag{B.23}$$

$$D_s^* = \frac{1}{1 + \alpha_s} \, ((1 + \alpha_s) S^* - P_s^* + \psi(P_n^* - P_s^*)) \tag{B.24}$$

$$\Pi_M^* = (w^* - c) D_n^* \tag{B.25}$$

$$\Pi_R^* = m^* D_n^* + (P_s^* - c) D_s^* - \frac{1}{2} k (S^*)^2. \tag{B.26}$$

$\square$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors made substantial contributions to this paper. Yongrui Duan developed the original idea and provided guidance. Zhixin Mao designed the game and calculated the process. Jiazhen Huo provided additional guidance and advice. All authors have read and approved the final manuscript.

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